The Rates of Evaporation of Sprays

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This paper reports a computational study of the evaporation rates of sprays of pure liquid drops having negligible velocity with respect to the air, and for drop velocities great enough to affect the evaporation rate. The principle parameters considered were the mean diameter and distribution of drop sizes in the initial spray, ratio of air to spray, initial temperature difference between air and spray, initial drop velocity, and air velocity.

The process of evaporation from sprays of liquid droplets into hot air is important in spray drying as well as in the combustion of liquid fuels. Such evaporation is a complex phenomenon because sprays normally contain droplets of a wide range of sizes, each with a constantly changing velocity and rate of evaporation. Many studies have been made on the rates of evaporation from single drops of pure liquids, and on the distribution of drop sizes produced by common atomizing devices. The correlations proposed as a result of these studies make it possible to write the equations governing the evaporation from sprays of drops, by taking into account the variation of conditions with both time and drop size.

The purpose of this investigation was to develop the mathematical equations for the evaporation of sprays with nonuniform drop size distribution and to solve them numerically for cases of interest. The investigation was limited to drops of pure liquid that remain at constant drop temperature with ideal cocurrent or countercurrent air flow.

The problem was divided into two cases. Case I treated sprays which have sufficiently low relative velocities, so that the rate of evaporation corresponded to zero velocity conditions. The calculations for this case covered a wide range of applicable independent parameters expressed in reduced form. In Case II sprays of appreciable relative velocity were treated. Because of a much greater complexity of these calculations and the larger number of independent parameters involved, calculations were made only for a selected number of the possible cases.

IMPORTANT RESULTS AND CONCLUSIONS

The investigation of sprays at low velocities showed that:

1. Sprays with less uniform drop size distributions evaporate more rapidly in the initial interval of time than more uniform sprays with the same mean diameter, because the many smaller drops initially present evaporate at higher rates. However the less uniform sprays take much longer for complete evaporation since there are more large drops which evaporate slowly.
2. Unless the air rate is infinitely great, the air temperature falls as the spray evaporates, and the rate of evaporation decreases.
3. Greatly different lengths of time are required for the evaporation of sprays which have different distributions but the same Sauter mean diameter. No mean diameter can adequately characterize a nonuniform spray with respect to its evaporative behavior. Rather, the distribution of drop sizes must be taken into account.
4. The size distribution of drops changes during evaporation. There is a general tendency for the average diameter of the remaining drops to increase in the moderately or highly nonuniform sprays and to decrease in the more uniform sprays.

When the evaporation of sprays with appreciable velocity is compared to evaporation at zero velocity with other conditions the same, the following occurs:

1. The distance traveled by the spray to achieve a given degree of evaporation is much greater, especially at the start.
2. For a given high initial velocity, the relative error in neglecting the velocity of the drops is greatest for small drops. The smaller drops decelerate during an appreciable fraction of the distance which they travel during evaporation.

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3. The velocity effect is more significant at higher initial velocities and at higher temperature differentials.

4. If small drops have initial velocities of the magnitude usually produced by common atomizing devices, the usual assumption that they will reach terminal velocity almost instantaneously is not generally valid in the calculation of the distance that such drops will travel during evaporation.

5. The shift in the instantaneous drop distribution during the evaporation is similar to that at low velocity.

The calculational approach with the use of a high speed computer is a valuable technique in the study of the evaporation of sprays. However, this approach cannot be applied to the study of evaporation under physical conditions much more complex than those assumed in this study, since quantitative starting equations cannot be written for many of the phenomena which one would like to take into account. Such effects include the presence of solids in the drops, back mixing of air, interference among drops, and transient behavior during and immediately after the formation of the drops. Calculations of the type made in this study could, however, be extended to cover a wider range of conditions in the evaporation of sprays of appreciable velocity and to a consideration of drops whose temperature is not constant.

DISCUSSION:

CASE I. EVAPORATION OF SPRAYS AT LOW VELOCITIES: THEORETICAL DEVELOPMENT

Assumptions and Conditions

Since the actual situation existing in a spray dryer or other process involving the evaporation of a spray is complex, it is necessary to describe a model or ideal spray process which can be expressed mathematically and yet approximates actual physical conditions. This development therefore assumes the following conditions:

1. The drops are spherical and sufficiently separated so that they fall and evaporate independently, that is, each as a single drop.
2. The drops are of pure liquid. This assumption is probably the greatest limitation in applying these results to commercial spray drying.
3. The dryer operates adiabatically with no heat losses.
4. The drops remain at a constant temperature. For air-water systems with which spray drying is primarily concerned, the drop temperature is, for all practical purposes, the same as the adiabatic saturation temperature.
5. The air moves in plug flow, and its temperature and humidity are uniform over the cross section. This involves the assumptions that there is no back mixing of the air, and the spray is spread uniformly over the air path.
6. The velocity (Reynolds number) of the drops with respect to the air is sufficiently low so that it does not affect the rate of heat transfer.

Basic Equations for the Evaporation of a Spray

Since it is assumed that the drop temperature is constant, all the heat transferred to the drop surface is used to evaporate liquid. The rate of evaporation of a single drop is written

$$-\frac{d}{d\theta}(1/6\,\pi\rho_L x^3) = h\,(\pi x^2)(T-t)/\lambda \qquad (1)$$

The heat transfer to the surface of an evaporating drop is less than without evaporation because some heat is absorbed in superheating the vapor as it diffuses out from the drop. It has been shown by Marshall (2) that this correction for superheat may be expressed at low velocities by the following:

$$N_{Nu} = \frac{hx}{k} = 2\left[\frac{\ln(1 + C_w(T-t)/\lambda)}{C_w(T-t)/\lambda}\right] \qquad (2)$$

The volume distribution of drop sizes in the initial spray is given by a normalized volumetric frequency function $\phi_V(x_1)$, such that $\phi_V dx_1$ is equal to the fraction of the total volume of the spray which is contained in drops having diameters between x_1 and $x_1 + dx_1$. If x is the instantaneous diameter of a drop having an initial diameter x_1, then the volume or mass fraction of the spray remaining at any time is

$$F = \int_0^\infty (x/x_1)^3 \phi_V dx_1 \qquad (3)$$

For a unit mass of initial spray, a material balance gives

$$wH_1 + 1 = wH + F \qquad (4)$$

Similarly a heat balance, referred to air and saturated vapor at temperature, t, gives

$$wC_{s1}\,(T_1 - t) + (-\lambda) = wC_s\,(T-t) + F\,(-\lambda) \quad (5)$$

The subscript, 1, refers to the location where the spray is admitted.

The air rate, w, is taken as positive for cocurrent flow and negative for countercurrent flow. Countercurrent flow is not strictly compatible with a zero relative velocity between the drops and the air. However, the assumption made requires only that this velocity be sufficiently small so as not to affect the rate of heat transfer.

Statement of Problem in Reduced Variables

The mathematics of this problem is simplified and the number of independent parameters to be considered is reduced by expressing it in terms of dimensionless independent parameters (designated by subscript o), and dimensionless dependent variables (designated by superscript *). The evaporation index, y, is introduced as an independent variable in place of time.

In terms of these dimensionless quantities Equation (1) becomes

$$d(x^*)^2/dy = -1 \qquad (6)$$

Therefore, for drops which have not completely evaporated we write

$$(x^*)^2 = (x_1^*)^2 - y, \quad x > 0 \qquad (7)$$

The diameter can thus be expressed as a function of its initial diameter and of a single independent variable, y, which is the same for all drop sizes.

Equations (3) and (5) become

$$F = \int_0^\infty (x^*/x_1^*)^3\,\phi_V^* dx_1^* \qquad (8)$$

and

$$C^*T^* = 1 - L_o + L_oF \qquad (9)$$

From the definitions of θ^* and y we obtain

$$\frac{d\theta^*}{dy} = \frac{1}{k^*T^*}\left[\frac{C_oT_oT^*}{\ln(1 + C_oT_oT^*)}\right] \qquad (10)$$

The thermal conductivity of air is proportional to the 0.84 power of the absolute temperature (5). Therefore, if k is evaluated at the arithmetic mean film temperature we obtain:

$$k^* = k/k_o = \left[\frac{1}{2}(T+t)\right]^{0.84} \Big/ t^{0.84} = (1 + T_oT^*)^{0.84} \qquad (11)$$

If the heat capacities of the air and vapor are assumed constant with temperature:

$$C^* = C_S/C_{S_1} = [C_{S_1} + C_w(H - H_1)]/C_{S_1} \quad (12)$$

On substituting $(H - H_1)$ from Equation (4), we get

$$C^* = 1 + C_w(1 - F)/C_{S_1}\, w = 1 + C_oT_oL_o(1 - F) \quad (13)$$

By the above set of equations θ^*, T^*, k^*, and C^* are expressed as functions of F; F as an integral involving the instantaneous diameters; and the instantaneous diameters as a function of the evaporation index, y, which is taken as the independent variable in this problem. These reduced quantities have the following significance:

y is the fractional reduction in surface area of a drop of initial diameter x_o. It is also equal to the reduced time, θ^*, if the air temperature is constant and if the correction for vapor superheating and the variation of k and C_S are neglected. As such, the evaporation index, y, is useful for expressing the changes of the properties of a given spray during evaporation independently of L_o.

T^* is the temperature driving force as a fraction of that existing at the point of introduction of the spray.

θ^* is the ratio of time to the time required for the complete evaporation of a drop of diameter x_o under a constant temperature difference $(T_1 - t)$.

L_o is the ratio of the heat of vaporization of the spray to the sensible heat of the air measured above its adiabatic saturation temperature at the point of introduction of the spray. Negative values of L_o denote countercurrent flow; positive values, cocurrent. L_o is zero for an infinite air rate and constant air temperature. It is unity for the minimum air rate necessary to completely evaporate the spray.

The conditions for any given problem in reduced form are specified by the parameters L_o, T_o, and C_o, and by the type of distribution for drop size in the initial spray. T_o and C_o are of only minor importance in varying the conditions of a problem since they come in only by expressing the relatively small variation of k and C_S and in the generally small correction for superheating the vapor. For the calculations discussed in this paper, T_o was taken as 0.3 and C_o as 0.528; these are the values for evaporation of water sprays with a drop temperature of 140°F. and an air temperature of 500°F., which are typical of spray drying.

The following four types of initial drop size distributions were considered; they are expressed in reduced form.

Log-normal:

$$\phi_V^* = \frac{1}{\sqrt{2\pi}\ \sigma\ x_1^*} \exp\left[-\frac{(\ln\ x_1^*)^2}{2\sigma^2}\right] \quad (14)$$

Square-root-normal:

$$\phi_V^* = \frac{1}{\sqrt{2\pi}\ 2\sigma^*\sqrt{x_1^*}} \exp\left[\frac{(\sqrt{x_1^*} - 1)^2}{2(\sigma^*)^2}\right] \quad (15)$$

Rosin-Rammler:

$$\phi_V^* = \delta(x_1^*)^{\delta-1} \exp[-(x_1^*)^\delta] \quad (16)$$

Nukiyama-Tanasawa:

$$\phi_V^* = \frac{\delta(5/\delta)^{6/\delta}\ (x_1^*)^5}{\Gamma(6/\delta)} \exp\left[-(5/\delta)(x_1^*)^\delta\right] \quad (17)$$

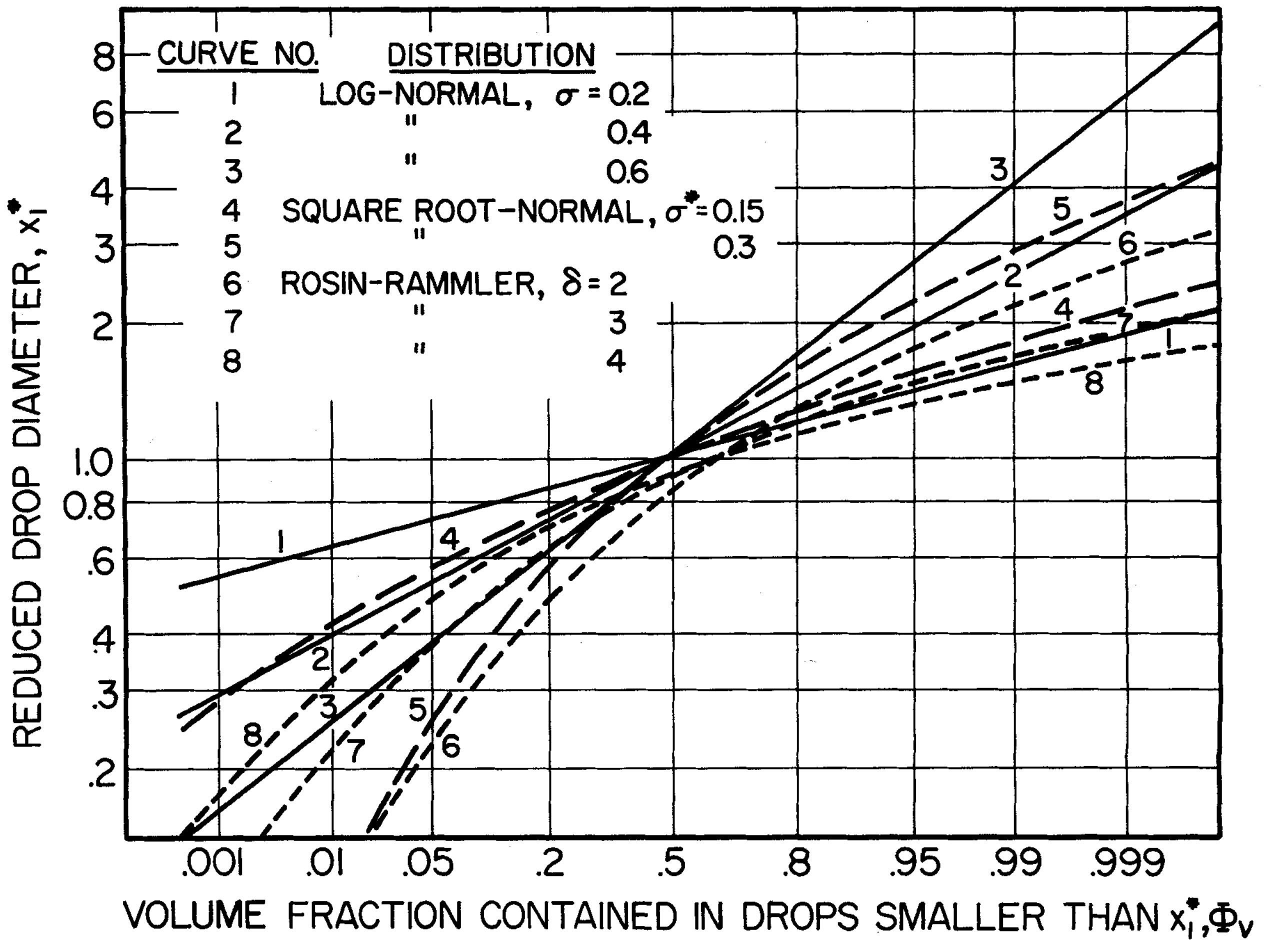

Fig. 1. Comparison of four different initial drop size distributions.

Each of these four equations contains a single parameter expressing the degree of dispersion of drop sizes about the mean. The cumulative functions of these distributions are compared in Figure 1.

Instantaneous Size Distribution

At a given time during the evaporation, the drop sizes will have a distribution different from that of the initial spray. The instantaneous frequency function per unit volume of initial spray, ϕ_v, is related to the initial frequency function, ϕ_V, by

$$\phi_v = (x/x_1)^4 \phi_V, \quad x \geq 0 \tag{18}$$

The fraction of the spray remaining unevaporated, Equation (3), thus may be written alternately in terms of the instantaneous distribution:

$$F = \int_0^\infty \phi_v \, dx \tag{19}$$

It is of interest to consider what initial distribution is required so that the normalized instantaneous frequency function will remain constant with time and be a function only of x. This problem was solved by Dickinson (*1*); when Equation (6) holds for a low velocity spray, the required function is

$$\phi_V^* = \frac{2}{3\sqrt{2\pi}} (x_1^*)^4 \exp\left[-(x_1^*)^2/2\right] \tag{20}$$

Calculational Procedure

It is not possible to evaluate the integral for F analytically. A Gaussian integration was performed, as follows. Equation (8) is transformed to an integral over a finite interval by the change of variable

$$du = \phi_V^* dx_1^* \tag{21}$$

with

$$u = -\frac{1}{2} \quad \text{at} \quad x_1^* = 0$$

Then by application of Gauss' formula for 15 ordinates

$$F = \sum_{i=1}^{15} R_i \, (x_i^*/x_{1_i}^*)^3 \tag{22}$$

where x_i^* is the instantaneous diameter of a drop of initial diameter, $x_{1_i}^*$; $x_{1_i}^*$ corresponds to u_i as given by solution of Equation (21) for a particular distribution; u_i is an ordinate in Gauss' formula (a fixed constant); R_i is the weighting coefficient for u_i (also a fixed constant).

Calculations were performed on a digital computer. First F was calculated as a function of y using Equations (7) and (22). Then using these data, θ^* was calculated as a function of F for different values of the parameter L_o by numerical integration of Equation (10). To study the change of the size distribution with time, F was evaluated in a few instances by numerical integration of Equation (19). Although the latter is much more time-consuming than the Gaussian integration, the instantaneous cumulative distribution Φ_v, is obtained at each step. Thus

$$\Phi_v(x^*) = \int_0^{x^*} \phi_v^* \, dx^* \tag{23}$$

PRESENTATION AND DISCUSSION OF RESULTS OF CALCULATIONS

Variation of Fraction Unevaporated

The effect of the drop size distribution on the rate of evaporation of a spray is best shown by graphs of the volume fraction of spray remaining unevaporated, F, on a logarithmic scale as a function of the evaporation index, y. It will be remembered that y can be interpreted as a reduced time, referred to the time to evaporate a drop of

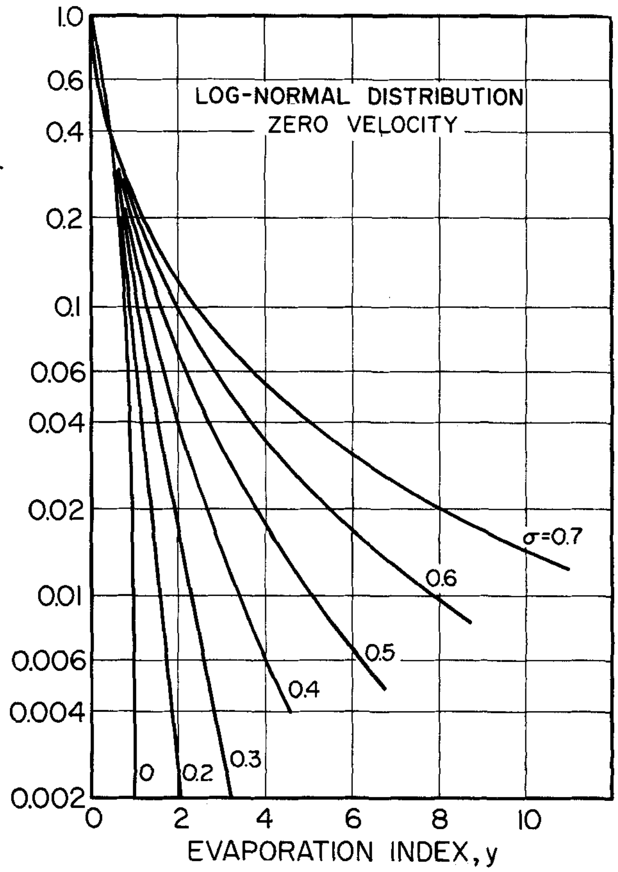

Fig. 2. Fraction unevaporated, F vs. evaporation index, y.

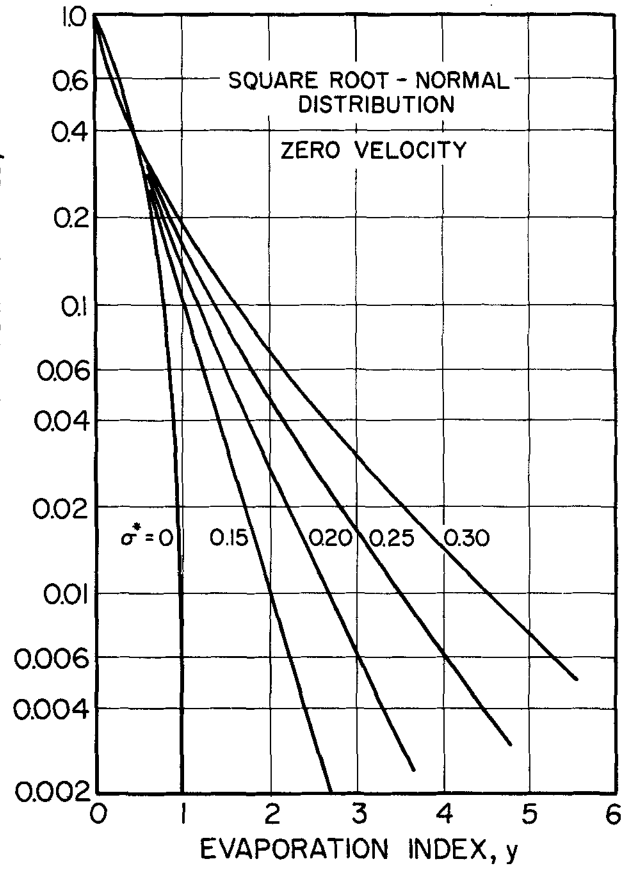

Fig. 3. Fraction unevaporated, F vs. evaporation index, y.

diameter of x_0 under the ideal condition of constant air temperature. For low velocity sprays this variation of F with y is determined only by a single parameter, (σ, σ^*, or δ), expressing the degree of dispersion of drop sizes about the mean. Such graphs at different values of the dispersion parameter, are given in Figures 2, 3, 4 and 5 for the log-normal, square-root normal, Rosin-Rammler, and Nukiyama-Tanasawa distributions.

The initial evaporation rate is greater for the less uniform distributions than for the more uniform, as the larger number of very small drops evaporates rapidly. However, the evaporation rate in the later stages is lower for the less uniform sprays because of the presence of more of the large drops which evaporate slowly.

The results for the Rosin-Rammler distribution agree with the calculations of the same type made by Probert (*4*).

The effect of L_0 (the ratio of latent heat of the spray to sensible heat of the air) is shown in Figure 6; graphs of F against θ^*, on logarithmic scales, are plotted for the log-normal distribution with σ of 0.4. The effect of L_0 on the evaporation time is great. The higher L_0, the greater is the reduction in the temperature driving force and hence in the evaporation rate. In the limiting case when L_0 is 1.0, the air temperature approaches the drop temperature as F approaches zero, and the driving force similarly approaches zero; the time for evaporation is thereby greatly increased. For countercurrent flow L_0 is negative, and the time of evaporation is considerably less than for cocurrent flow with the same air temperature at the point of introduction of the spray.

The reduced time required for the evaporation of 99% of the spray is given in Table 1 as a function of L_0 and of the distribution. Again, the time required is much greater for higher L_0 and for the less uniform distributions.

In design problems the time required for the complete evaporation of a spray is of interest. Since the distributions considered in this study had no maximum drop sizes, the time for their complete evaporation is infinite. The volume assigned to the larger drops by the distribution equations used is very small and the distributions of large sizes predicted by these equations are not unrealistic, even though they have no upper limits. The time required for the virtually complete evaporation of a spray depends

TABLE 1. TIME REQUIRED FOR 99% EVAPORATION
Values of Reduced Time, θ^*

for Case *I* $F = 0.01$, $T_o = 0.3$, $C_o = 0.528$

L_o =		0.0	0.4	0.6	0.8	0.9	1.0
Distribution							
ln	$\sigma = 0.2$	1.38	2.19	3.06	5.16	8.2	34
	0.3	1.99	3.29	4.73	8.32	13.7	64
	0.4	2.95	5.01	7.34	13.25	22.5	112
	0.5	4.45	7.70	11.44	21.16	36.5	191
	0.6	6.69	11.74	17.63	33.10	57.8	312
	0.7	10.06	17.85	26.97	51.23	90.4	497
srn	$\sigma^* = 0.15$	1.74	2.84	4.05	6.98	11.3	50
	0.20	2.29	3.82	5.52	9.77	16.2	74
	0.25	2.99	5.06	7.39	13.25	22.2	104
	0.30	3.88	6.64	9.80	17.80	30.1	147
rr	$\delta = 2.0$	2.22	3.78	5.52	9.91	16.5	76
	2.5	1.74	2.90	4.18	7.36	12.1	52
	3.0	1.50	2.46	3.51	6.11	9.9	44
	3.5	1.33	2.15	3.04	5.21	8.4	35
	4.0	1.22	1.95	2.74	4.64	7.4	30
nt	$\delta = 0.5$	9.35	16.32	24.15	44.67	77.0	401
	1.0	3.30	5.54	8.05	14.35	23.9	110

ln = log-normal
srn = square-root normal
rr = Rosin-Rammler
nt = Nukiyama-Tanasawa

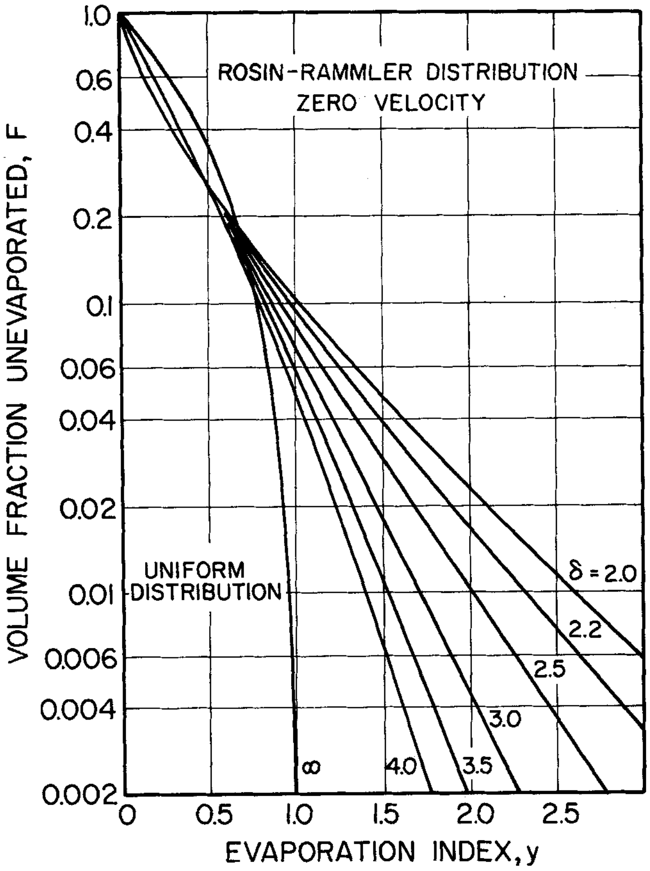

Fig. 4. Fraction unevaporated, *F* vs. evaporation index, *y*.

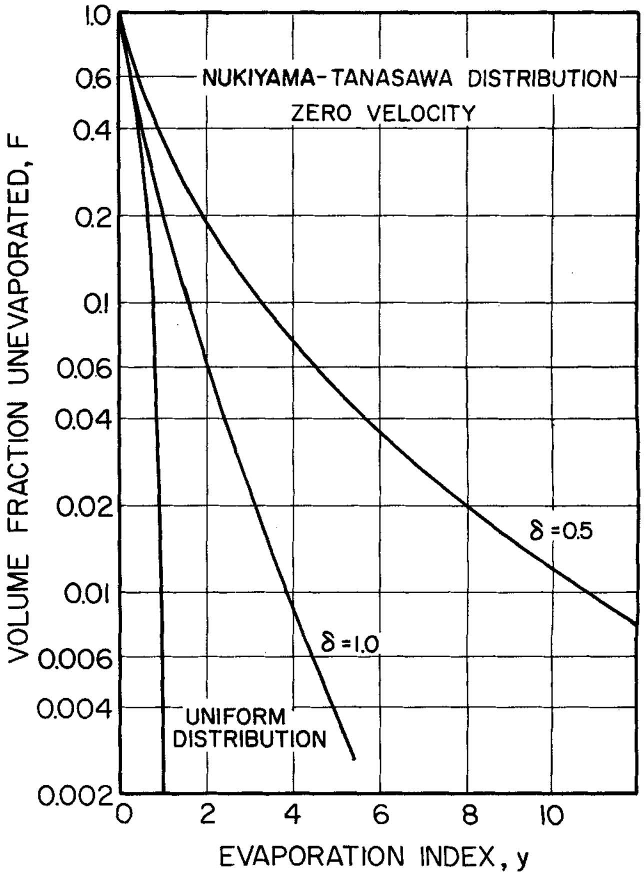

Fig. 5. Fraction unevaporated, *F* vs. evaporation index, *y*.

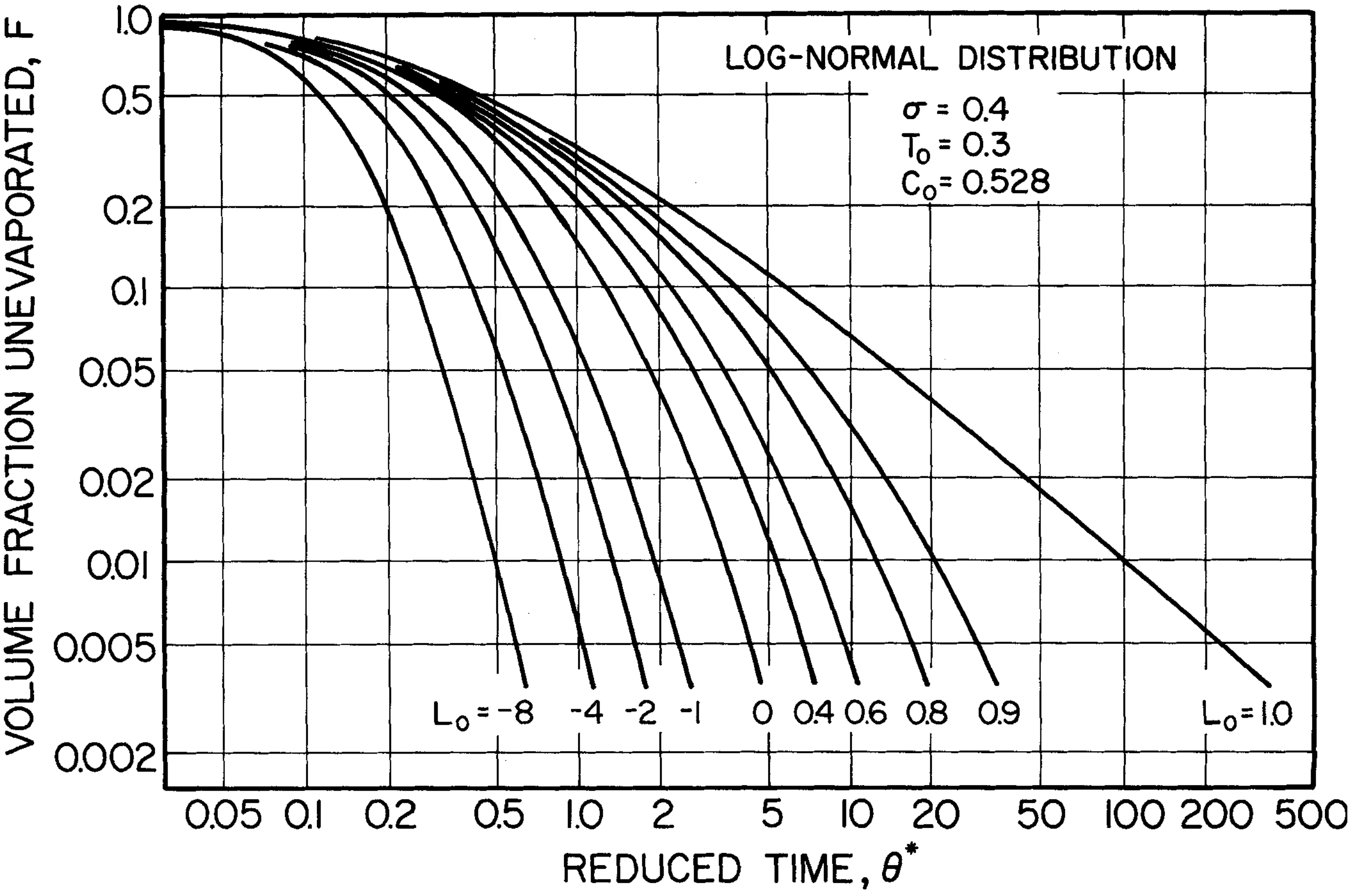

Fig. 6. Variation of fraction unevaporated with time as a function of L_0.

principally on the size distribution of the very large drops. Unfortunately, complete data on the distribution of the very large drops are seldom obtained in experimental size counts because of the statistical infrequency of these large sizes.

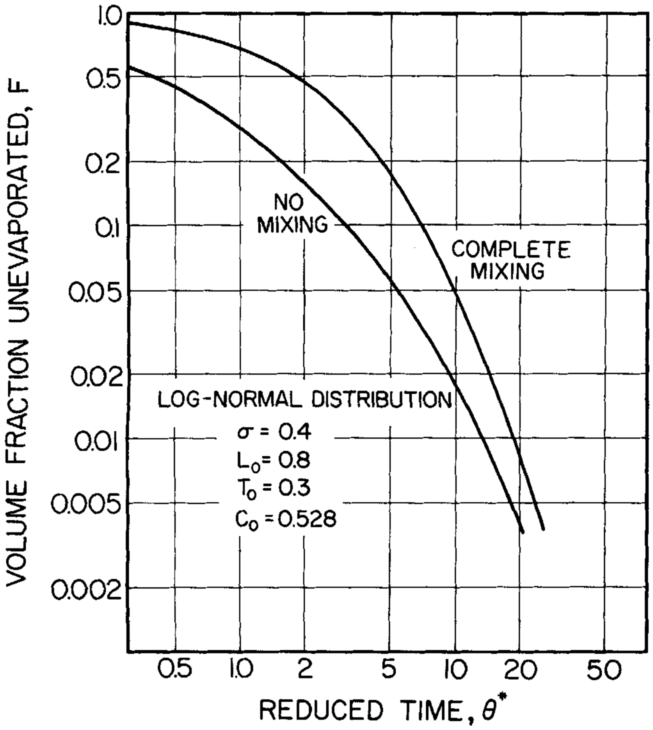

Fig. 7. Effect of back mixing of air.

Effect of Back Mixing of Air

In an actual spray dryer there is considerable back mixing or vertical recirculation of the air rather than the straight line flow assumed in these calculations. Although the exact degree and nature of this back mixing are usually not known and are difficult to take into account in calculations of evaporation time, the limiting case of complete mixing of the air is easily solved since the temperature and humidity of the air are then uniform at their outlet values, which are established by overall heat and material balances. Graphs of θ^* as a function of F are shown in Figure 7 for complete mixing and for no mixing, for L_o = 0.8.

When mixing takes place, the initial rate of evaporation is much less because the temperature difference is less and a longer time is required to reach a given degree of evaporation. The relative increase in the evaporation time introduced by mixing is greatest at the initial stages of the evaporation. When the air temperature approaches the outlet temperature, which is independent of mixing, the rate of evaporation also approaches independence of the degree of mixing.

Comparison of Sprays Having Same Sauter-Mean Diameter

Since the Sauter-mean diameter, $\bar{x}_{32}$, is often used to characterize a nonuniform spray with respect to its behavior during evaporation, it is interesting to compare the evaporation of several sprays following different drop size distributions but having the same Sauter-mean diameter. Figure 8 shows the behavior of several sprays each with a Sauter-mean diameter of 50 μ. The fraction remaining unevaporated is plotted as a function of time.

The evaporation rates of these different distributions differ greatly. The time required for 90% evaporation

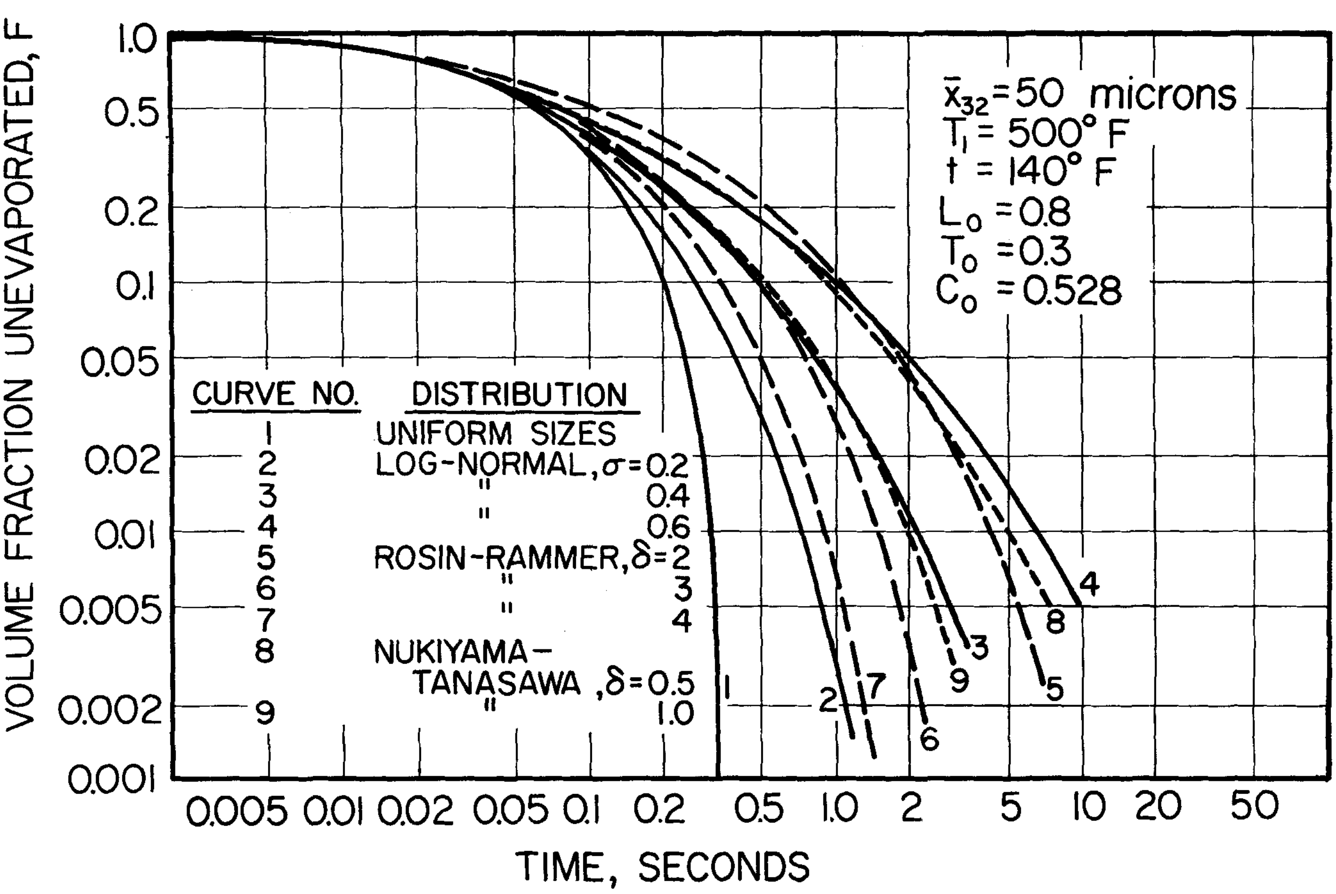

Fig. 8. Comparison of evaporation of sprays having same Sauter mean diameter.

varies by a factor of 5; that for 99% evaporation varies by a factor of 22. In general, the less uniform sprays require a longer time for evaporation; the uniform distribution requires the least. It is clear from a comparison of these graphs that the Sauter-mean diameter is a poor predictor of the evaporation time of a nonuniform spray.

It is also evident that no single mean diameter can properly characterize the evaporation of a spray. The selection of any other definition of an effective mean diameter would result only in a change in the value of x_0, at which different sprays are compared, and this would result only in a horizontal shifting of these curves as drawn on a logarithmic scale. Although the graphs for different distributions may be made to coincide at any desired value of F, they will not coincide at other values. Therefore, not only the mean diameter but also the distribution of drop sizes about this mean must be specified in order to predict the behavior of a spray during evaporation.

Variation of Drop Size Distributions During Evaporation

Figures 9 through 14 show the variation of the normalized instantaneous cumulative drop size distribution function, $\Phi_v(x^*)$, during the course of the evaporation, for several different initial distributions. The manner in which Φ_v shifts with time depends on the nature of the distribution, and few generalizations can be made.

The Rosin-Rammler distribution with $\delta = 2$, Figure 9, is characterized by a large number of small drops. As these evaporate there is a reduction in the fraction of the spray contained in the smaller drops. That is, the elimination of the smaller drops by their complete evaporation predominates over the reduction in size of those drops which are not completely evaporated, and the distribution is shifted toward the larger sizes. The cumulative distribution function, Φ_v thus decreases at all values of x^*, and the mass median diameter increases. This shift of

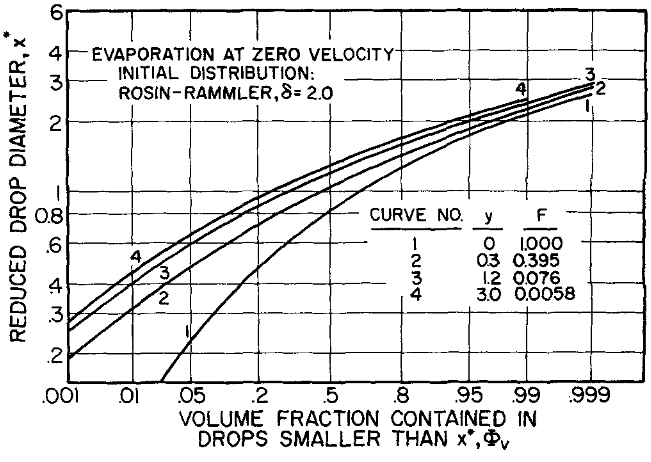

Fig. 9. Change of drop size distribution.

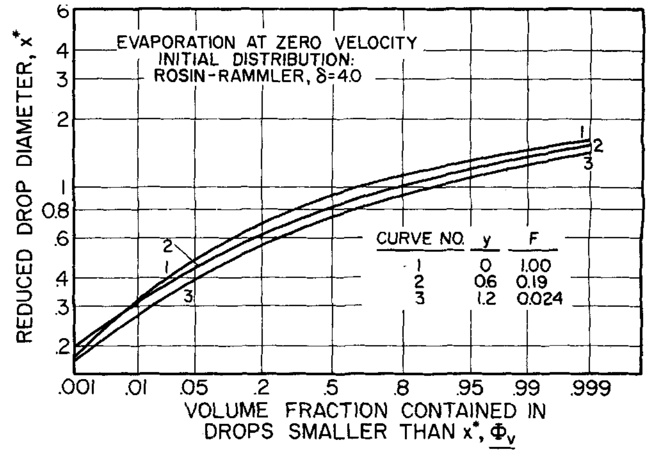

Fig. 10. Change of drop size distribution.

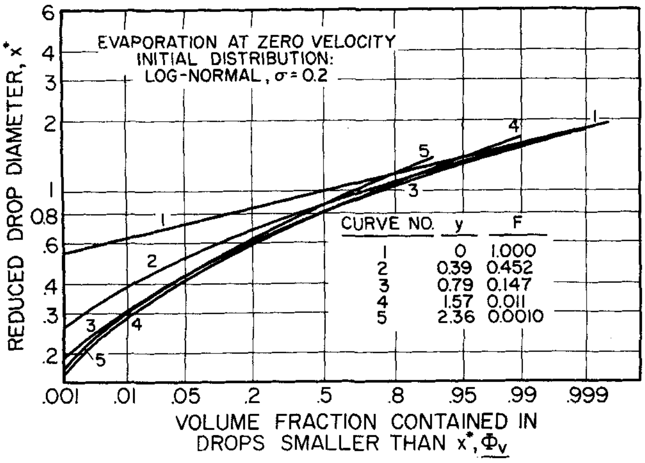

Fig. 11. Change of drop size distribution.

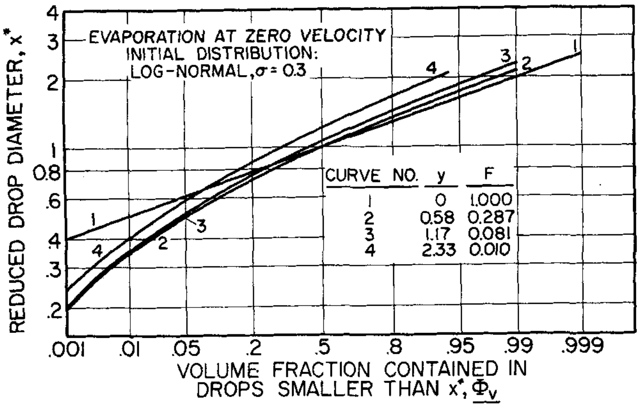

Fig. 12. Change of drop size distribution.

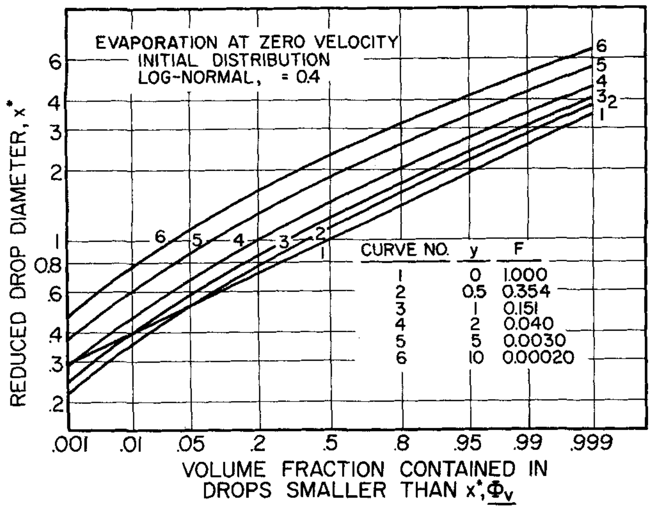

Fig. 13. Change of drop size distribution.

the distribution to larger sizes is encountered throughout the evaporation, but is particularly pronounced for the smallest drops during the first interval of time.

For the Rosin-Rammler distribution with $\delta = 4$, Figure 10, the result is the opposite. This distribution is more uniform, and the mass median diameter steadily decreases as all drops are reduced in size.

In the log-normal distributions there are a great number of the very large drops as compared to other distributions. Therefore, toward the end of the evaporation as the smaller drops disappear and the very large ones predominate, the distribution is shifted toward the larger sizes. During the initial interval of the evaporation there is some increase in Φ_v for at least the very smallest values of x^* as the more prevalent next larger classes are reduced in size; for the more uniform distributions this effect is much greater and persists a longer time and into larger drop sizes. Thus for $\sigma = 0.2$ (uniform) the mass median diameter initially falls, then rises slowly; for $\sigma = 0.3$, it initially remains constant, then rises; for $\sigma = 0.4$ or 0.5 (nonuniform) it increases continually, as shown in Figures 11 through 14.

The variation of the reduced Sauter-mean diameter, $\bar{x}_{32}^*$, is shown in Figure 15. The behavior is similar to that of the mass median diameter. Since none of these distributions have maximum drop sizes, it is not necessary that the mean diameter eventually fall to zero.

There is a general tendency for the instantaneous distribution to shift toward the larger sizes; that is, for the mean drop size to rise, in the moderately or highly non-

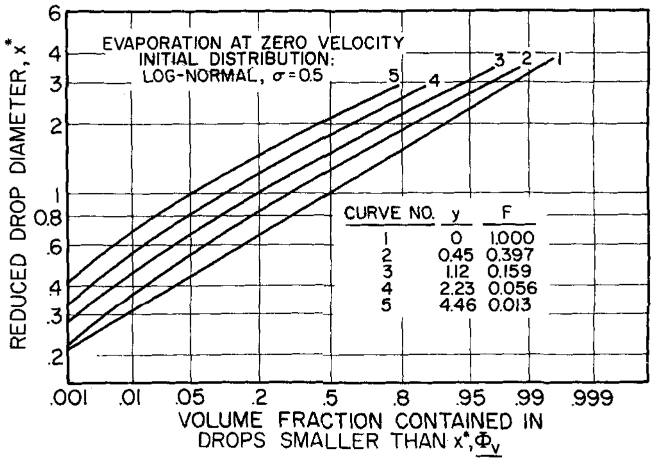

Fig. 14. Change of drop size distribution.

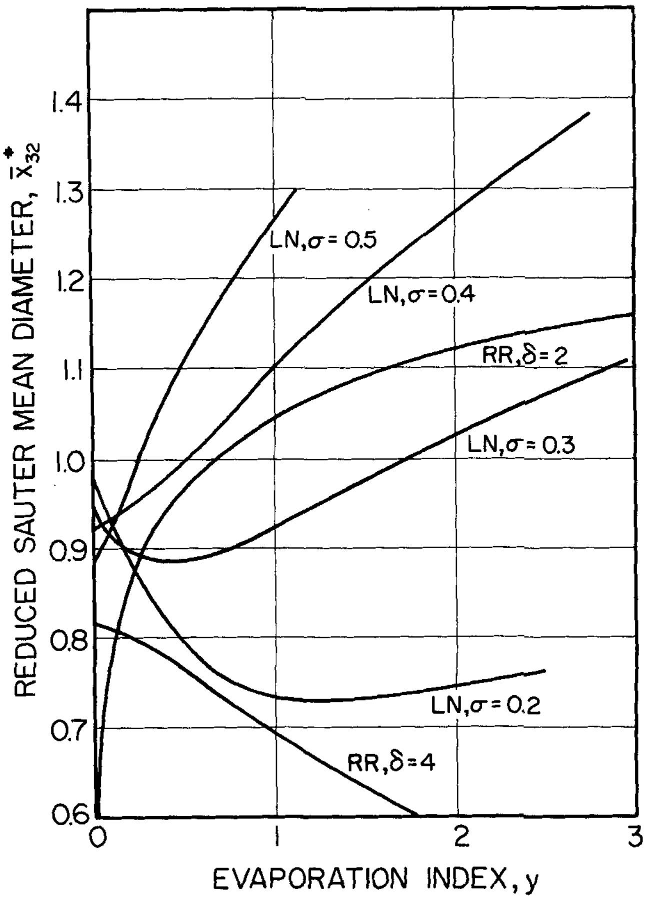

Fig. 15. Variation of Sauter mean diameter during evaporation at zero velocity.

uniform sprays and to shift toward smaller diameters, at least initially, in the sprays of more uniform sizes. In general the upper ends of the graphs of Φ_v tend to remain parallel to that of the initial distribution as the exaporation progresses, while the lower ends tend to approach the slope of the graph for that distribution for which Φ_v remains constant, [see Equation (20)].

If the distribution is such that Φ_v remains constant with time, then at constant air temperature the rate of evaporation per unit volume of remaining spray also remains constant. Therefore, for such a distribution, ln F is a linear function of y. A comparison of the variation of Φ_v and of ln F with y verifies this prediction. If for a given initial distribution the functional dependence of Φ_v on x^* changes but slightly over a range of y, then ln F is found to be nearly a linear function of y over this range. If the instantaneous distribution shifts toward the larger drops, as is true for the less uniform distributions, then $-d \ln F/dy$ decreases with increasing y, that is, the fractional evaporation rate becomes less as the mean drop size increases. If the opposite is true, $-d \ln F/dy$ increases. Thus, we may draw qualitative conclusions as to the variation of Φ_v with time if the variation of ln F with y is known for that distribution.

CASE II. DROPS WITH APPRECIABLE VELOCITY

In Case II the analysis for Case I is applied to the evaporation of sprays of drops which have velocities large enough to significantly affect their rates of evaporation. Large drop velocities make calculations of evaporation much more difficult. The differential equation for the rate of evaporation from a single drop can no longer be solved analytically. A second differential equation must be introduced to account for the variation of the drop velocity. Finally, the number of independent parameters required to specify a problem is increased.

Mathematical Statement of Problem

To simplify the calculations for Case II the humid heat, conductivity, viscosity, and density of the air were assumed to remain constant at some average values, and superheat of the vapor was neglected.

Heat transfer to a moving drop may be expressed by the correlation proposed by Ranz and Marshall (5).

$$N_{Nu} = 2(1 + 0.30\, N_{Pr}^{1/3}\, N_{Re}^{1/2}) \tag{24}$$

Equation (24) together with Equation (1) gives the rate of change of size of each drop. The deceleration of the drop is a function of gravity and drag. If the velocity is vertical (positive direction downward) we obtain

$$\frac{dv}{d\theta} = g - \frac{3\,\rho\, V^2\, C_D}{4\,\rho_L\, x} \tag{25}$$

Equations (3), (4), and (5) remain valid to evaluate F, H, and T. Since the drops do not all have the same velocity, we will take distance rather than time as the independent variable, and substitute

$$ds = (u + V)\, d\theta$$

The reduced notation used for Case I is retained with the exception that θ^* and y are redefined as:

$$\theta^* = 8k(T_1 - t)s/\lambda\rho_L x_o^2 u \tag{26}$$

$$dy = 8k(T - t)\, ds/\lambda\rho_L x_o^2 u \tag{27}$$

$$y = 0 \quad \text{at} \quad s = 0$$

It should be noted that θ^* is now a reduced distance rather than a reduced time. For Case I, $s/u = \theta$, and the definitions given in Case I for θ^* and y, if written for constant k and without the correction for mass transfer, are special cases of the above definitions.

In terms of this reduced notation taking y as the independent variable, the equations requiring solution become:

$$F = \int_o^\infty (x^*/x_1^*)^3\, \phi_V^*\, dx_1^* \tag{28}$$

$$T^* = 1 - L_o + L_o F \tag{29}$$

$$N_{Re} = (x_o\rho/\mu)x^* V \tag{30}$$

$$C_D = 0.22 + \frac{24}{N_{Re}}(1 + 0.15\, N_{Re}^{0.6}) \tag{31}$$

$$\frac{d(x^{*2})}{dy} = -\frac{1 + 0.30\, N_{Pr}^{1/3} N_{Re}^{1/2}}{1 + V/u} \tag{32}$$

$$\frac{dV}{dy} = \frac{A(B - V^2\ C_D/x^*)}{T^*(1 + V/u)} \tag{33}$$

where

$$A = 3\,\lambda\rho x_o/32k(T_1 - t)$$

$$B = 4\,\rho_L\, g x_o/3\rho$$

and

$$d\theta^*/dy = 1/T^* \tag{34}$$

Equation (31) was obtained by empirical fit of drag coefficients of solid spheres; it is valid within 7% for Reynolds numbers less than 3,000. The factor, $1 + V/u$, in Equations (32) and (33) arises from the definition of y as a reduced distance rather than time. Equations (28), (29), and (34) apply to the spray as a whole; the other four apply to each individual drop.

The same Gaussian integration to evaluate F [Equation (28)] was used as in Case I. This integral is over x_1^*

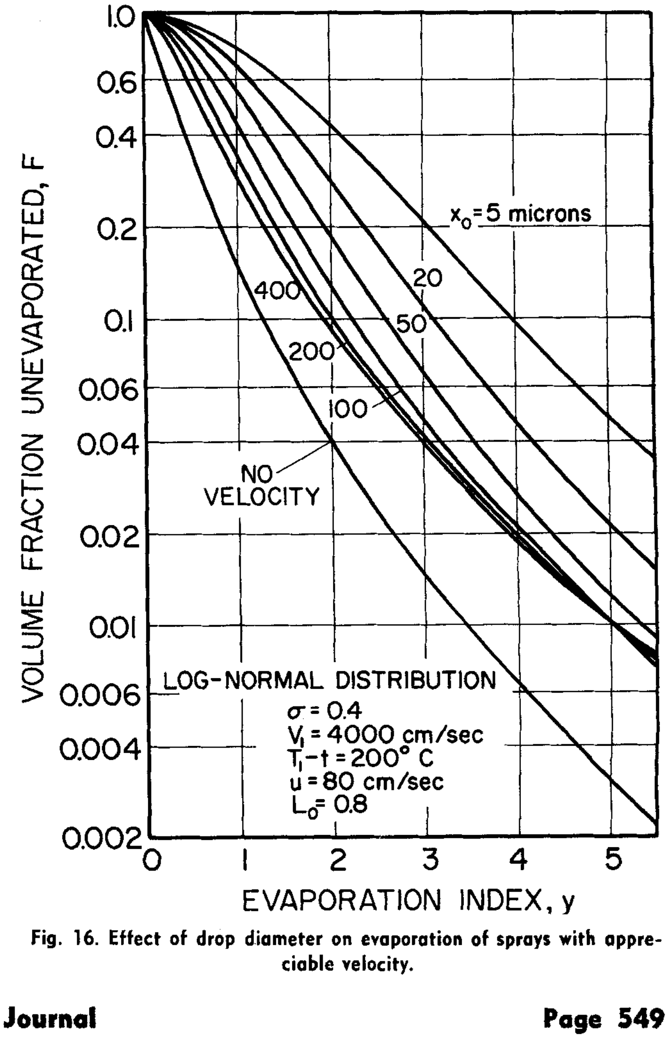

Fig. 16. Effect of drop diameter on evaporation of sprays with appreciable velocity.

rather than x^* and the same values of x_1^* are used each time the integration is repeated at a new value of y. Thus the diameter and velocity of only a limited number of drop sizes need be followed through the evaporation by solution of the differential Equations (32) and (33). In fact, this procedure is equivalent to considering 15 initial drop diameters, x_{1i}, with volume fractions, R_i, of each. Thus Gauss' integration formula provides a good means of approximating a continuous step-by-step distribution.

An expression for the initial reduced frequency function, ϕ_V^*, is also required to complete this set of equations. Since it was necessary to limit the calculations to a reasonable number, all calculations for Case II were made for the log-normal distribution with $\sigma = 0.4$ [Equation (14)]. The physical properties of the drops and air were taken as constant at their values for water drops at 60°C. and a mean air film temperature of 120°C.

Results of Calculations

The more significant results of the solutions of these equations are presented as graphs of the volume fraction remaining unevaporated, F, on a logarithmic scale, as a function of the evaporation index, y. Such graphs are shown in Figures 16 to 19; each figure shows the effect of the variation of a single parameter. At constant air temperature, y is a reduced distance; specifically it is the ratio of the distance which the spray has traveled from its point of injection to the distance which a drop of initial diameter, x_o, would travel during its complete evaporation, if it had zero velocity with respect to the air.

The corresponding graph for zero velocity (Case I) is shown on each figure for comparison. For Case I, the relation between F and y depends only on the initial distribution. Therefore, the single graph of F as a function of y is comparable to all the high velocity conditions shown in Figures 16 to 19. The results of Case II are best interpreted by comparing the distance required for a given degree of evaporation with appreciable velocity to that for zero velocity. It should be noted that y contains $(T_1 - t)/x_o^2u$ as a factor. Therefore, when comparing the results for different values of $(T_1 - t)$, x_o, or u, y is not proportional to the distance traveled by the spray.

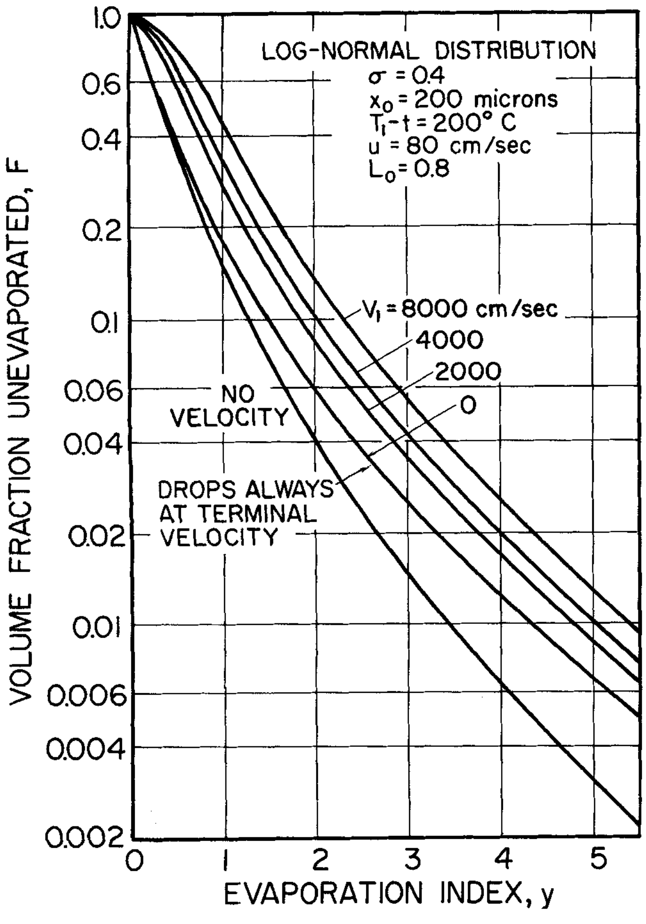

Fig. 17. Effect of initial velocity on evaporation of sprays with appreciable velocity.

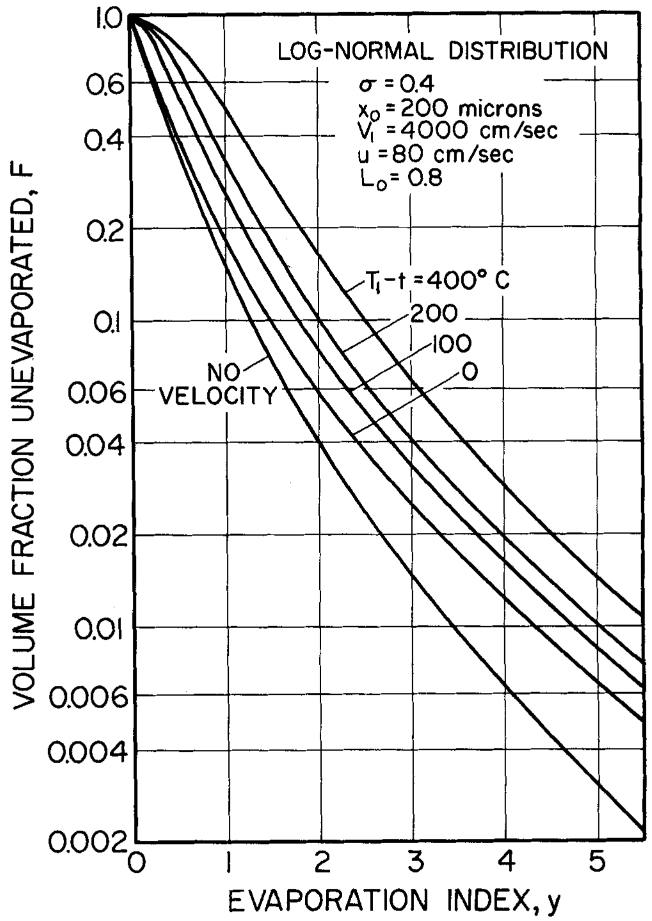

Fig. 18. Effect of initial temperature difference on evaporation of sprays with appreciable velocity.

In all instances, y is considerably greater, at the same F, for sprays with appreciable initial velocity than for those at zero velocity. The rate of evaporation increases only as the square root of velocity [as expressed by the numerator of Equation (32)] while the distance traveled increases with the first power (as expressed by the denominator of this equation). Thus the high velocity spray undergoes less evaporation per unit distance traveled.

Figure 16 shows the effect of variation of x_o. As x_o becomes smaller, the results become more different from those at zero velocity. For 90% evaporation ($F = 0.1$), y is 1.53 times that for zero velocity with x_o of 400 μ; 3.14 times at 5 μ. In other words, for drops with the same high initial velocity, the effect of velocity becomes more pronounced the smaller the drop. As the drop size is reduced, both the distance required for evaporation and that required for deceleration are reduced. However, the relative reduction in the former is greater. A small drop therefore spends a greater fraction of its life in deceleration, and its average velocity during its life is greater than that of a larger drop.

As x_o increases, its effect on the graphs of F vs. y becomes less, and these graphs meet and cross at low F. The larger drops have higher terminal velocity and therefore travel farther during the post deceleration stage of evaporation.

Figure 17 shows the effect of the initial velocity of the spray. As would be expected, the deviation from the case of zero velocity becomes greater as V_1 is increased. The rate of change of F with y is very dependent on V_1 at the start of the evaporation, but nearly independent of it at the end where terminal velocities have been reached or approached. If the drops are always at terminal velocity ($V_1 = 0$) the results are closer to those for Case I than to the results for high initial velocities. For example, at 90% evaporation ($F = 0.1$) y has values of 1.30, 1.50, and 2.03, respectively, for drops at zero velocity, drops always at terminal velocity, and drops with V_1 of 4,000 cm./sec. For small values of y the graphs for zero velocity and for drops always terminal velocity nearly coincide. In this region most of the evaporation takes place from the small drops whose terminal velocities are sufficiently low that $d(x^{*2})dy$ is nearly equal to -1.0, its value for Case I. At a higher y more of the evaporation takes place from larger drops whose terminal velocity is appreciable. It should be noted that the average size of the remaining drops increases during the course of the evaporation for the particular size distribution considered. For sprays having initial velocities of 1,000 cm./sec. or greater, the assumption that all drops instantaneously reach terminal velocity is a poor one in calculations of this type.

Figure 18 shows the effect of the initial temperature difference between the air and the spray. At higher temperature differences the rate of evaporation is greater, while the rate of deceleration remains the same. Therefore, at high temperature difference deceleration occupies a larger fraction of the lifetime of each drop, and the effect of the initial velocity is more pronounced. The effect of increased temperature difference is thus the same as that of increased initial velocity.

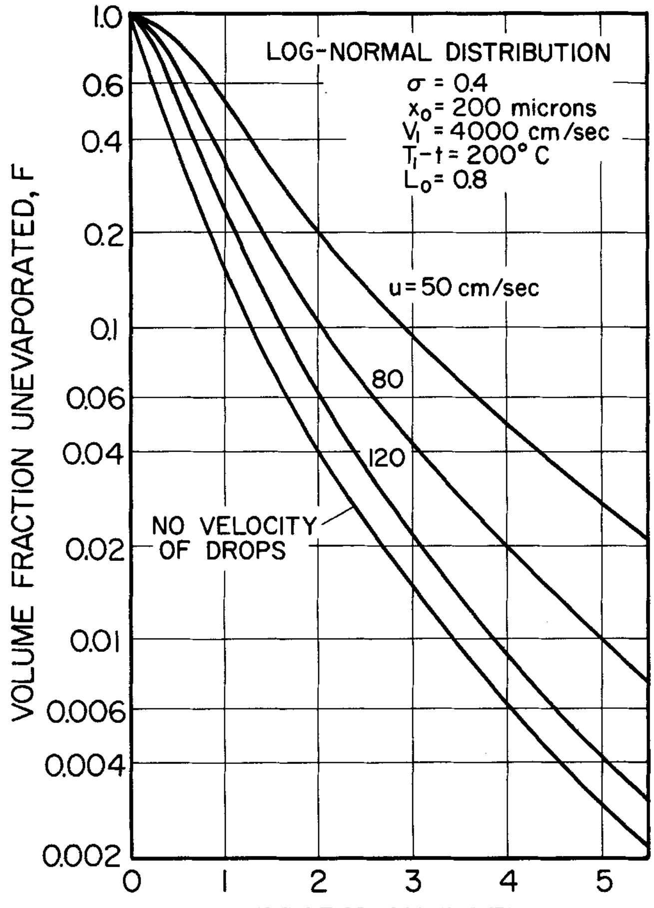

Fig. 19. Effect of absolute air velocity on evaporation of sprays with appreciable velocity.

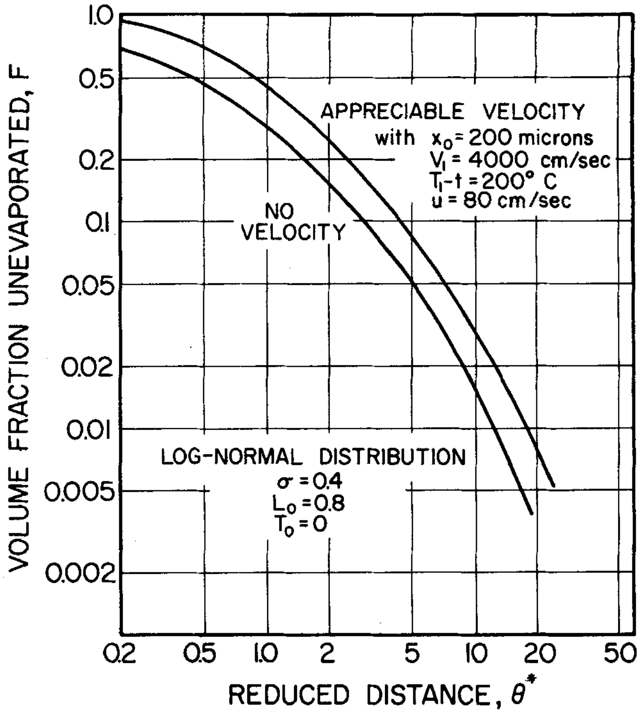

Fig. 20. Effect of drop velocity on distance required for evaporation.

Figure 19 shows the effect of the air velocity. At the higher air velocities the drop velocity is relatively less important and the results are closer to those of Case I. The air velocities considered in Figure 19 are of the same order of magnitude as the terminal velocity of the drops in the distribution considered.

Figure 20 shows a graph of F as a function of reduced distance, θ^*, for $L_o = 0.8$, as compared to the corresponding conditions for Case I. The qualitative difference between the two curves is about the same as for those of F as a function of y. This graph of F vs. reduced distance is similar in general shape to those calculated by Priem (3) for the evaporation of log-normal sprays of heptane in a rocket combustion chamber, a physical situation somewhat different from that of the present study. Priem also found the same qualitative effect of the variation of x_o as in Figure 16.

Figure 21 shows the variation of the cumulative drop size distribution function, Φ_v, during the course of the evaporation. The resulting instantaneous distributions are about the same as those for Case I at the same values of y (shown in Figure 13), except that there are fewer small drops. In Case II, $-d(x^*)^2/dy$ is considerably greater for the smaller drops than for the larger ones since they have lower velocities. Therefore, the smaller drops evaporate more rapidly as compared to the larger ones than is true in Case I, where $d(x^*)^2/dy$ is constant for all sizes. This results in an instantaneous distribution with fewer small drops than in Case I.

Also included in Figure 21 are lines connecting the points representing drops of constant initial diameter. These show the manner in which one cumulative distribution is transformed into the succeeding ones.

The distance required for individual drops to reach terminal velocity in these sprays is of interest. Under typical conditions for Case II (those of Figure 16, $x_o = 200\ \mu$) each drop lost about 24% of its volume during deceleration to terminal velocity. The largest and smallest of the 15 drop classes considered reached terminal velocity (234 and 14 cm./sec.) at values of the evaporation index, y, of 18 and 56%, respectively, of those required for complete evaporation of that drop. The quantity $d(x^*)^2/dy$, which expresses the rate of evaporotion per unit distance, was commonly −0.1 to −0.3 for decelerating drops but −0.9 to −1.0 for drops at terminal velocity. The evaporation, per unit of distance traveled, is therefore predominantly from a narrow size range of drops which have reached terminal velocity but not completely evaporated.

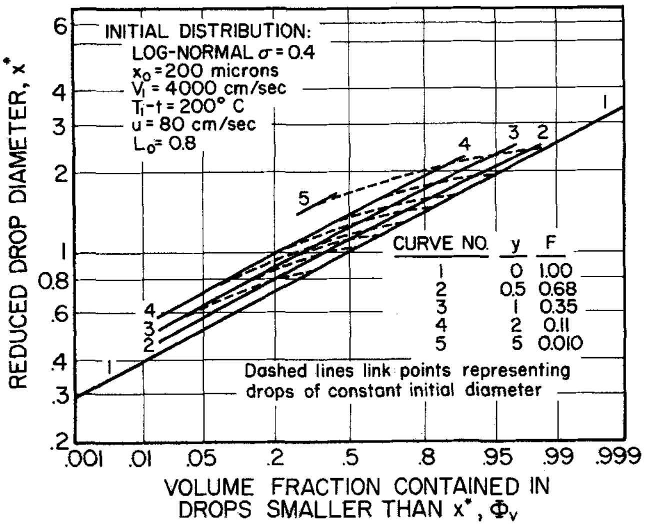

Fig. 21. Change of drop size distribution during evaporation at appreciable velocity.

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NOTATION

C_D = drag coefficient for sphere
C_o = $2t\,C_w/\lambda$
C_S = humid heat of moist air
C_w = heat capacity of vapor at constant pressure, per unit mass
C^* = C_S/C_{S1}
F = volume fraction of spray remaining unevaporated, per unit initial volume
g = gravitational acceleration
h = coefficient of heat transfer from bulk air steam to to drop surface based on surface area of drop
H = absolute humidity of air
k = thermal conductivity of air
k_o = thermal conductivity of air, evaluated at t
k^* = k/k_o
L_o = $\lambda/wC_{S1}(T_1 - t)$
N_{Nu} = Nusselt number for heat transfer = hx/k
N_{Pr} = Prandtl number for air
N_{Re} = Reynolds number = $xV\rho/\mu$
s = distance traveled by spray, measured downward from point of introduction
t = absolute temperature of drop
T = absolute temperature of bulk air stream
T_o = $(T_1 - t)/2t$
T^* = $(T - t)/(T_1 - t)$
u = linear velocity of air stream, positive downward, assumed constant
V = velocity of drop relative to air, positive downward
w = air rate, mass of dry air/unit mass of initial spray; positive for cocurrent flow, negative for countercurrent
x = drop diameter
x_1 = initial drop diameter
x_o = a reference or mean diameter appearing in distribution equation
x^* = x/x_o; similarly with this superscript on the other x's
y = evaporation index, a reduced time or distance referred to the ideal conditions of constant air temperature and zero drop velocity

$$= \int_o^\theta \frac{8k(T-t)\,d\theta}{\lambda\rho_L x_o^2}\left[\frac{\ln[1 + C_w(T-t)/\lambda]}{C_w(T-t)/\lambda}\right] \quad \text{(in Case I)}$$

$$= \int_o^s \frac{8k(T-t)\,ds}{\lambda\rho_L x_o^2 u} \quad \text{(in Case II)}$$

Greek Letters

$\Gamma(a)$ = gamma function = $\int_o^\infty t^{a-1}\underline{e}^{-t}dt$
δ = parameter in Rosin-Rammler and Nukiyama-Tanasawa distributions expressing uniformity of drop sizes
θ = time measured from time of introduction of drop
θ^* = reduced time or distance
$8k_o(T_1 - t)\theta/\lambda\rho_L x_o^2$ (in Case I)
$8k(T_1 - t)s/\lambda\rho_L x_o^2 u$ (in Case II)
λ = heat of vaporization of liquid, at t
μ = absolute viscosity of air
ρ = density of air
ρ_L = density of liquid
σ = standard deviation, a measure of the spread of drop sizes about the mean in normal type of distribution equations
σ^* = $\sigma/\sqrt{x_o}$ for square-root-normal distribution
ϕ = drop size frequency function = $d\Phi/dx$
ϕ^* = $x_o\phi$
Φ = cumulative drop size distribution function

Superscripts

$*$ = reduced quantity (made dimensionless)
underline of ϕ or Φ denotes a distribution which has been normalized; that is

$$\underline{\phi} = \frac{\phi}{\int_o^\infty \phi\,dx}$$

Subscripts

1 = initial conditions or conditions at point of introduction of spray
o = dimensionless independent parameter
V = on Φ or ϕ denote initial distribution with respect to volume
v = denotes the corresponding instantaneous distributions

LITERATURE CITED

1. Dickinson, D. R., Ph.D. dissertation, Univ. of Wisconsin, Madison (1958).
2. Marshall, W. R., Jr., *Trans. Am. Soc. Mech. Engrs.*, **77**, 1377 (1955).
3. Priem, R. J., *Nat. Advisory Comm. Aeron., Tech. Note*, 4098 (1957).
4. Probert R. P., *Phil. Mag.* **37**, 94 (1946).
5. Ranz, W. E., and W. R. Marshall, Jr., *Chem. Engr. Prog.*, **48**, 141, 173 (1952).

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